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Cubic Casimir operator of $\mathrm{SU}_C(3)$ and confinement in the nonrelativistic quark model

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Abstract

Only two-body $[\mathrm{F}_i \cdot \mathrm{F}_j]$ confining potentials have been considered, thus far, in the quark model without gluons, which by construction can only depend on the quadratic Casimir operator of the colour SU(3) group. A three-quark potential that depends on the cubic Casimir operator is added to the quark model. This results in improved properties of q^3 colour non-singlet states, which can now be arranged to have (arbitrarily) higher energy than the singlet, and the "colour dissolution/anticonfinement" problem of the $\mathrm{F}_i \cdot \mathrm{F}_j$ model is avoided.

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I. INTRODUCTION

In spite of the wide-spread concensus on the validity of QCD as the theory of strong interactions, QCD has proven essentially intractable, except in perturbative approximations. [1] It is fair to say that little (or no) understanding of quark confinement has been achieved since QCD's inception more than 25 years ago [2]

Instead of solving QCD one often resorts to various forms of the quark model, perhaps the simplest version being the nonrelativistic (n.r.) constituent quark model [3]. The spin-statistics problem of the simplest quark model led to the introduction of the colour degrees of freedom that obey the SU(3) Lie algebra (this led subsequently to QCD). This "colour SU(3)" is exactly conserved (there is no colour leakage), hence the quark model spectrum must fall into irreducible representations of this group, and the quark Hamiltonian must be expressible in terms of SU(3) invariant operators. There are two such independent "invariants" of SU(3), the so-called Casimir operators, other than the unit operator [2,3].

Even with an infinitely rising ("confining") $q - q$ potential, that does not allow separation of individual quarks from their aggregates, in the simplest quark model, which assumes colour independent quark interactions, the coloured q^3 states are degenerate with the colour-singlet one. This is in manifest contradiction with the experience. To remedy this shortcoming, a colour-dependent factor $\mathrm{F}_i \cdot \mathrm{F}_j$ (proportional to the colour charges of the two quarks $\mathrm{F}^a_{i,j}$) was introduced into the two-quark potential of the quark model, in analogy with the one-gluon exchange (OGE) potential in QCD. This goes by the name of the "$\mathrm{F}_i \cdot \mathrm{F}_j$ model". This colour factor is proportional to the first ("quadratic") Casimir operator $C^{(1)} = \mathrm{F}^a\mathrm{F}^a = \mathrm{F} \cdot \mathrm{F} = \mathrm{F}^2$, where F^a are the group generators of SU(3). No three-quark or higher-order interactions have been allowed in this model so far. Whereas many view three- and many-body forces with distaste, it is also indisputable that QCD demands three-, and four-quark interactions at the tree approximation level. Therefore it ought to have been clear all along that no two-body interaction could describe QCD completely.

A. Problems with the $\mathrm{F}_i \cdot \mathrm{F}_j$ model

As noted by many, e.g. [4,5], the $\mathrm{F}_i \cdot \mathrm{F}_j$ model suffers from a number of weaknesses:

1. it predicts unstable coloured q^2 and $q\bar{q}$ states (the "colour dissolution / anticonfinement" problem);

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[1] The highest achievement of lattice QCD is the (mere) extraction of a linearly rising colour-singlet $q\bar{q}$ potential, which is far from proving physical confinement. In particular, the existence and energetics of coloured states remain unexplored. Moreover, multiple colour-singlet multi-quark states appear in the theory and it is unclear just which one lies lowest.

[2] In this regard, see M. Chanowitz's remarks made 23 years ago [1].

[3] One may think of it as QCD with gluon degrees of freedom frozen, the quark-quark interaction being transmitted by potentials of colour-exchange character.

2. it predicts towers of new, as yet undiscovered multiquark states, $q^2\bar{q}^2$ being the lowest lying ones (the "colour chemistry" problem);

3. it predicts unobserved long-range forces between colour singlets (the v.d. Waals force problem).

The standard "solution" to problem (1), the assumption that only colour singlet states exist, is entirely *ad hoc* and thus unsatisfactory. Moreover, it does not begin to address problems (2) and (3). Hence we shall seek a change in the dynamics of the quark model that might lead to the solution of the confinement problem(s). We consider the displacement of coloured states to (arbitrarily) high energies/masses as a solution to the confinement problem.

This note will show that the introduction of a three-quark force proportional to the second ("cubic") Casimir operator can fix (at least some of) these shortcomings. [4] To be sure, such a three-quark potential is not an arbitrary addition: it arises from the instanton-induced 't Hooft interaction in QCD. What we do assume, however, is that its spatial behaviour is confining, which is *ad hoc*.

II. THREE-BODY POTENTIAL

The three-quark potential can be factored into a colour part $\mathcal{C}_{123}$ and the spin-spatial part $\mathcal{V}_{123}$:

$$V_{123} = \mathcal{C}_{123}\mathcal{V}_{123}. \tag{1}$$

The following 3-body colour factors can be written down:

$$\mathcal{C}_{123} = \begin{cases} \sum_{i<j}^{3} \mathrm{F}_i \cdot \mathrm{F}_j = \mathrm{F}_1 \cdot \mathrm{F}_2 + \mathrm{F}_1 \cdot \mathrm{F}_3 + \mathrm{F}_2 \cdot \mathrm{F}_3 \\ d^{abc}\mathrm{F}_1^a\mathrm{F}_2^b\mathrm{F}_3^c \\ if^{abc}\mathrm{F}_1^a\mathrm{F}_2^b\mathrm{F}_3^c, \end{cases} \tag{2}$$

where $\mathrm{F}^a = \frac{1}{2}\lambda^a$ is the quark colour charge, the lower index indicates the number of the quark, λ^a are the Gell-Mann matrices, and f^{abc}, d^{abc} are the SU(3) structure constants. Only the first two factors, Eqs. (2,2) are SU(3) invariants, however, i.e., only they can be expressed in terms of Casimir operators as follows

$$\sum_{i<j}^{3} \mathrm{F}_i \cdot \mathrm{F}_j = \frac{1}{2}C^{(1)}_{i+j+k} - 2 \tag{3}$$

$$d^{abc}\mathrm{F}_1^a\mathrm{F}_2^b\mathrm{F}_3^c = \frac{1}{6}\left[C^{(2)}_{i+j+k} - \frac{5}{2}C^{(1)}_{i+j+k} + \frac{20}{3}\right]; \tag{4}$$

where $i+j+k$ stands for the three-quark colour state. Only the second factor, Eq. (4), depends on the cubic Casimir operator. The third colour factor, Eq. (2), is an off-diagonal

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[4] The second Casimir operator $C^{(2)}$ of SU(3) is tri-linear ("cubic") in the group generators F^a, *viz.* $C^{(2)} = d^{abc}\mathrm{F}^a\mathrm{F}^b\mathrm{F}^c$, so it can only appear in three- or more-quark potentials.

operator that annihilates the two SU(3) eigenstates with definite exchange symmetry, i.e. the **1** and **10**, see Table I, and converts one **8** state into another. Therefore, it is not allowed in the quark model Hamiltonian [5].

The first two colour factors Eqs. (2,2) are symmetric under the interchange of any pair of indices $i \leftrightarrow j$, $i \leftrightarrow k$, and $j \leftrightarrow k$, whereas the third one, Eq. (2), is antisymmetric. All three are symmetric under cyclic permutations $i \to j \to k$ and $i \to k \to j$. Since the complete potential has to be symmetric under each of these permutations, the corresponding spin-spatial parts have to have the same symmetry properties as the colour ones. Consequently, the third one has to be spin dependent, whereas the first two need not. For the sake of simplicity in this letter we limit ourselves to spin-independent potentials, i.e. again to the first two cases, Eqs. (2,2).

Keeping with the tradition of the quark model, we take the harmonic oscillator for both the two- and three-quark spatial parts of potentials:

$$\mathcal{V}_{12} = \frac{1}{2} m\omega^2 \left(\mathbf{r}_1 - \mathbf{r}_2\right)^2 \tag{5}$$

$$\mathcal{V}_{123} = c\frac{1}{2} m\omega^2 \left[\left(\mathbf{r}_1 - \mathbf{r}_2\right)^2 + \left(\mathbf{r}_3 - \mathbf{r}_2\right)^2 + \left(\mathbf{r}_1 - \mathbf{r}_3\right)^2\right]; \tag{6}$$

with an as yet undetermined strength c for the latter. With the harmonic oscillator assumption we find that the $\mathrm{F}_i \cdot \mathrm{F}_j$ model two-body interaction leads to the same form of the effective potential in the q^3 system as the three-body force Eq. (2). Similar statements hold for the colour-independent two- and three-body potentials. For this reason there is no need to introduce such two- and three-body potentials separately, but only one of a kind. We shall show that a colour-independent two-body potential is necessary for the stabilization of both $q\bar{q}$ and q^3 spectra. Hence we do not introduce a separate colour-independent three-body potential. With these results we can write down the Hamiltonians for few-quark systems and then solve for their spectra.

A. q^3 Hamiltonian and its spectrum

We shall start with the $\mathrm{F}_i \cdot \mathrm{F}_j$ model two-body potential and show that the "cubic Casimir" 3-body force alone cannot stabilize it. We find the following Hamiltonians in the colour channels of the q^3 system

$$H_1 = \sum_{i}^{3} \frac{\mathbf{p}_i^2}{2m} + \frac{2}{3}\sum_{i<j}^{3} \mathcal{V}_{ij} + \frac{10}{9}\mathcal{V}_{123} \tag{7}$$

$$H_8 = \sum_{i}^{3} \frac{\mathbf{p}_i^2}{2m} + \frac{1}{6}\sum_{i<j}^{3} \mathcal{V}_{ij} - \frac{5}{36}\mathcal{V}_{123} \tag{8}$$

$$H_{10} = \sum_{i}^{3} \frac{\mathbf{p}_i^2}{2m} - \frac{1}{3}\sum_{i<j}^{3} \mathcal{V}_{ij} + \frac{1}{9}\mathcal{V}_{123}. \tag{9}$$

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[5] It is true, of course, that the "Mercedes-Benz star" diagram of QCD carries this colour factor. This factor does not appear in the quark model because that diagram alone is not gauge invariant.

After going to centre-of-mass and Jacobi coordinates $\boldsymbol{\rho}, \boldsymbol{\lambda}$ we find the following potentials

$$V_1 = \left(\frac{2}{3} + \frac{10}{9}c\right) \frac{3}{2} m\omega^2 \left(\boldsymbol{\rho}^2 + \boldsymbol{\lambda}^2\right) \tag{10}$$

$$V_8 = \left(\frac{1}{6} - \frac{5}{36}c\right) \frac{3}{2} m\omega^2 \left(\boldsymbol{\rho}^2 + \boldsymbol{\lambda}^2\right) \tag{11}$$

$$V_{10} = \left(-\frac{1}{3} + \frac{1}{9}c\right) \frac{3}{2} m\omega^2 \left(\boldsymbol{\rho}^2 + \boldsymbol{\lambda}^2\right) \tag{12}$$

from which we can read off the stability conditions as

$$1 > -\frac{5}{3}c \tag{13}$$

$$1 > \frac{5}{6}c \tag{14}$$

$$1 < \frac{1}{3}c. \tag{15}$$

Note that two of the three inequalities are in conflict, regardless of the sign of c. We conclude that the cubic Casimir three-body force cannot stabilize the 3q system with the $\mathrm{F}_i \cdot \mathrm{F}_j$ model two-quark interaction. Consequently, we turn to modification of this model that will lead to stable states in both the $q\bar{q}$ and q^3 systems.

Sufficient condition for the stabilization of the **8** $q\bar{q}$ state is to have as the two-body potential

$$V_{12} = \left[c_1 + \frac{4}{3} + \mathrm{F}_i \cdot \mathrm{F}_j\right] \frac{1}{2} m\omega^2 \left(\mathbf{r}_1 - \mathbf{r}_2\right)^2, \tag{16}$$

with $c_1 > 0$. For simplicity we take $c_1 = 1$. With this two-body potential we find

$$V_1 = \left(\frac{5}{3} + \frac{10}{9}c\right) \frac{3}{2} m\omega^2 \left(\boldsymbol{\rho}^2 + \boldsymbol{\lambda}^2\right) \tag{17}$$

$$V_8 = \left(\frac{13}{6} - \frac{5}{36}c\right) \frac{3}{2} m\omega^2 \left(\boldsymbol{\rho}^2 + \boldsymbol{\lambda}^2\right) \tag{18}$$

$$V_{10} = \left(\frac{8}{3} + \frac{1}{9}c\right) \frac{3}{2} m\omega^2 \left(\boldsymbol{\rho}^2 + \boldsymbol{\lambda}^2\right) \tag{19}$$

Note that with this new and improved two-body interaction and *no* three-body force ($c = 0$), all three q^3 colour states are stable, but the octet **8** is lighter than the singlet **1**, again in contrast with the experiment!

Turning on the three-body force, $c \neq 0$, we find the following stability condition

$$-\frac{3}{2} < c < \frac{78}{5}. \tag{20}$$

For values of $c < \frac{2}{5}$ we find the anticipated ordering of colour states: singlet **1** is the lowest lying, the next lowest is the octet **8**, and then the decimet **10**. The ratio of their ground state energies can be made arbitrarily large by choosing c sufficiently close to - 1.5. For example, with $c =$ - 1.43, the **8** and **10** states are lying above 4 GeV. We conclude that the colour-independent two-body force stabilizes the q^3 system, whereas the cubic Casimir three-body force makes it well ordered in colour, i.e. properly confined.

This should not be a surprise: for the colour singlet to separate away from other colour multiplets, the Hamiltonian must contain at least one piece proportional to the colosinglet projection operator P_1 shown below

$$\mathrm{P}_1 = \frac{1}{27} - \frac{1}{36}\sum_{i<j}^{3} \boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j + \frac{1}{12} d^{abc} \boldsymbol{\lambda}_1^a \boldsymbol{\lambda}_2^b \boldsymbol{\lambda}_3^c \tag{21a}$$

$$\mathrm{P}_8 = \frac{16}{27} - \frac{1}{9}\sum_{i<j}^{3} \boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j - \frac{1}{6} d^{abc} \boldsymbol{\lambda}_1^a \boldsymbol{\lambda}_2^b \boldsymbol{\lambda}_3^c \tag{21b}$$

$$\mathrm{P}_{10} = \frac{10}{27} + \frac{5}{36}\sum_{i<j}^{3} \boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j + \frac{1}{12} d^{abc} \boldsymbol{\lambda}_1^a \boldsymbol{\lambda}_2^b \boldsymbol{\lambda}_3^c \tag{21c}$$

which manifestly depends on both the two-body and the three-body operators. Without the three-quark (cubic Casimir), there is bound to be some (accidental) degeneracy left and the colour singlet state could not be isolated.

Concrete (observable) phenomenological consequences of our new three-quark interaction only become visible in ("exotic") multiquark systems, such as $q^4\bar{q}$, or $q^2\bar{q}^2$, because the "ordinary" states, such as q^3, only allow one colour singlet, and non-singlet states have not been observed.

III. THE $Q^2\bar{Q}^2$ SYSTEM

Having found a confining potential that predicts the presumed ordering of the q^3 colour spectrum, we turn to its application to the $q^2\bar{q}^2$ system. We break up this system into three-body configurations. For our purposes one can equivalently think of this system either as $(q^2\bar{q})\bar{q}$, or as $q(q\bar{q}^2)$. Thus we need to evaluate the three-body potential's colour factor in variously coloured $q^2\bar{q}$ and $q\bar{q}^2$ states.

The $q^2\bar{q}$ system can occupy one of the following four colour states $(\mathbf{3}\otimes\mathbf{3})\otimes\bar{\mathbf{3}} = (\bar{\mathbf{3}}\oplus\mathbf{6})\otimes\bar{\mathbf{3}} = (\bar{\mathbf{6}}\oplus\mathbf{3}_a)\oplus(\mathbf{3}_s\oplus\mathbf{15})$. Note that there are two colour triplets and that they have different symmetry properties under the interchange of the two quark indices: one is symmetric, another antisymmetric. This means that there will be two colour singlet $q^2\bar{q}^2$ states with corresponding quark interchange symmetry properties. Our three-body interaction will distinguish between the two colour singlet states.

We must be careful about the definition of the colour factors in the nonrelativistic three-body potential involving antiquarks as they are sensitive to the C-conjugation properties of the relativistic interaction from which the potential was derived (the latter's properties carry over into the nonrelativistic limit for odd number of quarks). More specifically, one finds a difference between the Lorentz scalar and zeroth component of Lorentz vector models, which is unusual. In the quark model one ordinarily replaces the quark colour factor F^a by

$$\bar{\mathrm{F}}^a = -\frac{1}{2}\boldsymbol{\lambda}^{aT} = -\frac{1}{2}\boldsymbol{\lambda}^{a*} \tag{22}$$

which is the definition of the colour *charge* operator of an antiquark. Note, however, that the minus sign in this definition stems from the C-conjugation properties of the *vector* current and not from SU(3) itself. So for Lorentz scalar, pseudoscalar and axial-vector interactions this sign changes into a plus. [6] In two-body potentials this sign makes no difference, as there are two such factors that cancel. In the three-body case the sign makes a difference. For example, due to the odd number of interacting particles there is a sign difference between the "cubic Casimir" three-quark and three-antiquark potentials in the Lorentz-vector model (unlike the two-body potential), thus apparently violating C-conjugation symmetry and CPT, since P- and T- are conserved. For Lorentz scalar interactions the antiquark potential has the same sign as the quark potential and there is no such problem. Lorentz-vector 3-point functions are forbidden by C-conjugation ("Furry's theorem") anyway, so we conclude that only Lorentz scalar three-quark potential is allowed.

Thus we conclude that the "cubic Casimir" three-body interaction must have the following colour factor when antiquarks are involved

$$\bar{\mathcal{C}}_{123} = \begin{cases} -d^{abc}\mathrm{F}_1^a\mathrm{F}_2^b\bar{\mathrm{F}}_3^c \\ \ \ d^{abc}\mathrm{F}_1^a\bar{\mathrm{F}}_2^b\bar{\mathrm{F}}_3^c \\ -d^{abc}\bar{\mathrm{F}}_1^a\bar{\mathrm{F}}_2^b\bar{\mathrm{F}}_3^c \end{cases} \tag{23}$$

Once again, we can express the two SU(3) invariant colour factors in terms of the Casimir operators. The first one remains unchanged:

$$\sum_{i<j}^{3} \mathrm{F}_i \cdot \mathrm{F}_j = \mathrm{F}_1 \cdot \mathrm{F}_2 + \mathrm{F}_1 \cdot \bar{\mathrm{F}}_3 + \mathrm{F}_2 \cdot \bar{\mathrm{F}}_3 = \frac{1}{2}C^{(1)}_{i+j+k} - 2, \tag{24}$$

whereas the second one becomes

$$d^{abc}\mathrm{F}_1^a\mathrm{F}_2^b\bar{\mathrm{F}}_3^c = \frac{1}{6}\left[C^{(2)}_{i+j+k} - \frac{5}{2}C^{(1)}_{i+j} + \frac{50}{9}\right]. \tag{25}$$

Note that in the second factor, Eq. (25) the first (quadratic) Casimir is evaluated between the two-quark (sub-)state $i+j$, which leads to a distinction between the two overall colour triplets (which are symmetric and antisymmetric in the quark indices). This leads to results shown in Table II. Using Table II, we find the following potentials in the two overall colour singlet states $[s \equiv 8, a \equiv 1]$

$$V_s = c\frac{5}{18}\omega^2\left(\mathbf{r}_{12}^2 + \mathbf{r}_{13}^2 + \mathbf{r}_{14}^2 + \mathbf{r}_{23}^2 + \mathbf{r}_{24}^2 + \mathbf{r}_{34}^2\right) \tag{26}$$

$$V_a = -c\frac{5}{9}\omega^2\left(\mathbf{r}_{12}^2 + \mathbf{r}_{13}^2 + \mathbf{r}_{14}^2 + \mathbf{r}_{23}^2 + \mathbf{r}_{24}^2 + \mathbf{r}_{34}^2\right). \tag{27}$$

From the signs of the two interaction potentials we see that the mass/energy of the "octet" state is enhanced for $c \geq 0$ and vice versa for the "singlet" state. As we have already shown that c can be either positive or negative (it only needs to be less than 0.4), we conclude that the 3-body interaction can elevate the mass of the unobserved (symmetric) "octet" states above the conventional/ordinary two-meson states and thus make them less stable and less likely to be detected. In this sense, one may think of this three-body interaction as a solution to Isgur's problem ("fiasco") of unobserved (towers of) $q^2\bar{q}^2$ states in the $\mathrm{F}_i \cdot \mathrm{F}_j$ model.

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[6] This definition ignores the SU(3) analog of G-parity transformation.

IV. CONCLUSIONS

We have looked into the question of the second (cubic) Casimir interaction in the quark model and found that:

1. It is insufficient to stabilize the $\mathrm{F}_i \cdot \mathrm{F}_j$ model by itself. A colour-independent two-body force is necessary to prevent colour dissolution in both the $q\bar{q}$ and the q^3 systems.

2. In conjunction with a colour-independent two-body force it leads to a proper ordering in energy of the coloured q^3 states for three-body coupling constants c in the range $-1.5 < c < 0.4$.

3. In the $q^2\bar{q}^2$ system it leads to a distinction between the two colour singlet states, in that it enhances overall binding in one and diminishes it in the other, depending on the sign of its coupling constant c.

We have made several simplifying assumptions that can and ought to be relaxed in the future. For example: (1) harmonic oscillator nature-, and (2) spin independence of the three-body potential. Relaxation of these assumptions leads to new predictions in the observable (colour singlet) sector. For example, the replacement of the usual two-body "colour-spin" interaction with a more complicated, three-body one will change the pattern of SU(6) splitting in multi-quark states.

Many papers have been written on the "saturation" of quark-quark interactions, starting with those of Nambu [6] and of Greenberg and Zwanziger [7]. Those early papers assumed two-quark (quadratic Casimir operator) potentials that vanish at infinite quark-quark separations, in contrast with modern notions that allow them to infinitely rise. This work can be viewed as a natural continuation of those early works to models with infinitely rising potentials and three-quark (cubic Casimir) colour operators.

REFERENCES

[1] M. Chanowitz, p. 25 in *Color Symmetry and Quark Confinement, vol. III, Proceedings of the Twelfth Rencontre de Moriond conference*, editor J. Tran Thanh Van, R.M.I.E.M., Orsay, France (1978).

[2] G.E. Baird, and L.C. Biedenharn, Jour. Math. Phys. **4**, 436 (1963).

[3] A. Pais, Rev. Mod. Phys. **38**, 215 (1966).

[4] A. LeYaouanc, L. Oliver, O. Pène, and J.C. Raynal, Phys. Rev. D **42**, 3123 (1990).

[5] J. Weinstein, and N. Isgur, Phys. Rev. D **41**, 2236 (1990).

[6] Y. Nambu, in *Preludes in Theoretical Physics*, ed. A. de Shalit et al., North Holland, Amsterdam, (1966).

[7] O.W. Greenberg, and D. Zwanziger, Phys. Rev. **150**, 1177 (1966).

TABLES

TABLE I. Diagonal matrix elements of the three operators for variously coloured q^3 states. Of course, there are two distinct **8** states, but they are equivalent in this regard.

	1	**8**	**10**
$\langle\sum_{i<j}^{3} \mathrm{F}_i \cdot \mathrm{F}_j\rangle$	-2	- $\frac{1}{2}$	1
$\langle d^{abc}\mathrm{F}_1^a\mathrm{F}_2^b\mathrm{F}_3^c\rangle$	$\frac{10}{9}$	- $\frac{5}{36}$	$\frac{1}{9}$
$\langle f^{abc}\mathrm{F}_1^a\mathrm{F}_2^b\mathrm{F}_3^c\rangle$	0	0	0

TABLE II. Diagonal matrix elements of the three-body colour operators for variously coloured $q^2\bar{q}$ states.

	$\mathbf{3}_a$	$\mathbf{3}_s$	$\bar{\mathbf{6}}$	**15**
$\langle\sum_{i<j}^{3} \mathrm{F}_i \cdot \mathrm{F}_j\rangle$	-$\frac{4}{3}$	-$\frac{4}{3}$	- $\frac{1}{3}$	$\frac{2}{3}$
$\langle d^{abc}\mathrm{F}_1^a\mathrm{F}_2^b\bar{\mathrm{F}}_3^c\rangle$	$\frac{5}{9}$	- $\frac{5}{18}$	$-\frac{5}{18}$	$\frac{1}{18}$